Tutorial
Discrete Optimization - Centrale Supélec
Solution

F. Rocaries & H. Talbot

April 18, 2018

Here is a collection of problems in integer and binary programming.

1 A young couple

A youg couple, Alice et Benoît, constantly argue with respect to domestic chores (and they don't even have children yet!). In order to solve the problem, as good mathematicians, they try to find the most equitable way to share the chores. After a lot of research, they draw the following tableau:

	Groceries	Cooking	Dishwashing	Laundry
Alice	4.5h	7.8	3.6	2.9
Benoît	4.9h	7.2	4.3	3.1

They decide to each only take care of two tasks, which cannot be shared. They first decide to search for the most *efficient* solution, i.e. the one that minimizes their working hours.

1. Formulate this problem as an IP (Integer Program).
2. Solve the LP (Linear Program) relaxation, for example with our crude Simplex solver in Python. Don't forget to initialize the problem.
3. Comment on the solution as much as possible:
   - Qualify the solution
   - Why do we have degenerate basis variables in the final solution?
   - What is the relationship to the expected size of the basis ?
   - Any other comment?
4. No stranger to politics, they had decided that instead of having the most efficient workload, they should have the most *equitable* workload irrespective of efficiency. How would you formulate and solve the problem ?

2 Processors

A processor manufacturer wants to reinvent their product line. To help with their product line decision, the following tableau is given:

	Product 1	Product 2	Product 3	Product 4
Starting costs	€50 000	€40 000	€70 000	€60 000
Revenue/unit	€70	€60	€90	€80

Denoting x_i is the production of i, we want to maximize the profit (i.e. revenues - costs), knowing that:

- At most two products can be fabricated.
- Product 3 or 4 require the production of one of product 1 or 2.
- There exist production limits, expressed thus:

$$\begin{array}{rcrcrcrcll} 5x_1 & + & 3x_2 & + & 6x_3 & + & 4x_4 & \leq & 6000 & \text{or} \\ 4x_1 & + & 6x_2 & + & 3x_3 & + & 5x_4 & \leq & 6000 & \end{array}$$

1. Model the problem as a mixed linear-integer program.
2. Solve the problem with a spreadsheet program

3 The textbooks

Company WSP sells textbooks. This company has two sale representatives to best cover a region split into zones. The number of students in each zone is given in figure 1:

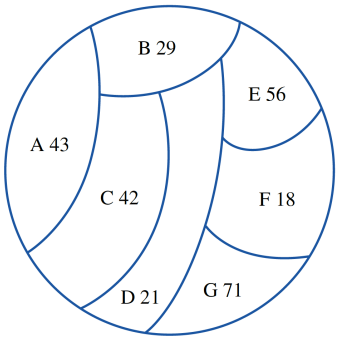

Figure 1: Régions

Each representative must be associated to two adjacent zones, for instance one rep can be affected to zones A and B but not A and D.

a Propose a formulation allowing WSP to maximize the number of students reached by the two representatives.

b Write and solve the problem with a spreadsheet program.