

IF4-ALG2 - TD Optimization

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1 Modern spreadsheet and solvers

Modern spreadsheet programs (Excel, OpenOffice or LibreOffice) all can solve Linear and Integer programs.

1.1 An example with OpenOffice

The screenshot are from the OpenOffice.org Wiki, available here:

https://wiki.openoffice.org/wiki/Optimization_Solver.

Refer to that documentation if needed.

We will formulate and solve the following problem with a spreadsheet program:

A company produces doors and windows. It has three production plants, together with time and resources constraints, formulated in the following way:

$$\begin{aligned} \max z = & 3x_1 + 2x_2 \\ & x_1 & \leq 4 \\ & 2x_2 & \leq 12 \\ & 3x_1 + 2x_2 & \leq 18 \end{aligned}$$

Of course the company wants to maximize its profit. We now use OpenOffice to solve this problem.

Start the solver Choose the menu item `Tools -> Solver...`. You should see a non-modal dialog window similar to the one on Fig. 1.

Non-modal means you can keep interacting with both the spreadsheet and the solver dialog.

Create the spreadsheet Fill a spreadsheet similar to the one on Fig. 2 :

Fill the constraints in the solver

1. Variables are in `B10` et `B11` ; formulas for the constraints in `B14–B19` ; the limits for the constraints are in `C14–C19` ; Finally objective/profit is in `$B5`.

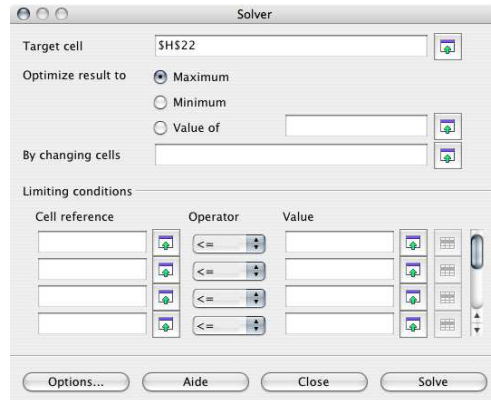


Figure 1: Solver dialog

	A	B	C
1		Portes et fenêtres	
2			
3			
4			
5	Objectif	0	
6			
7			
8	Variables		
9			
10	Portes/semaine	0	
11	Fenêtres/semaine	0	
12			
13	Contraintes		
14			
15	Usine 1	0	4
16	Usine 2	0	12
17	Usine 3	0	18
18	Non-neg 1	0	0
19	Non-neg2	0	0
20			
21			

Figure 2: Spreadsheet initialization

2. Specify the formula for the constraints, for instance, in \$B\$17 you may enter $=3 * \$B\$10 + 2 * \$B\11 , etc.
3. Set the objective at the “Target cell”
4. Set variables in “By changing cells”. You may specify them manually as \$B\$10:\$B\$11, ou select them with the mouse.
5. Enter the references for the constraint cells in the same way.
6. Press the “solve” button.

The procedure is basically the same for Excel. Report the optimum!

2 The auto mechanic

An auto mechanic is specialized in oil change and engine tuning. The profit per oil change is 7 euros. The profit per engine tuning is 15 euros. The mechanic has a contract with a car rental company that guarantees 30 oil changes per weeks. Each oil change requires 20 minutes of work and 8 euros of parts. An engine tuning requires 1h of work and 15 euros of parts. Workers are payed 10 euros an hour, and the mechanic employs two of them at 35h per week. Each week the mechanic buys 1750 euros of parts that can be used either for oil changes or engine tuning. The mechanic would like to maximize his profit.

- Formulate this problem as a linear program;
- Put is in standard form with the smallest possible number of variables;
- Solve this problem with the simplex method using a spreadsheet solver.

3 Alloys

A metal worker must produce 100kg of an alloy that is composed of 30% lead, 30% zinc and 40% tin. He must achieve this composition from pre-existing alloys existing on the open market. The following alloys are available:

Alliage	1	2	3	4	5	6	7	8	9
% Lead	10	10	40	60	30	30	30	50	20
% Zinc	10	30	50	30	30	40	20	40	30
% Tin	80	60	10	10	40	30	50	10	50
Cost per kg	4.1	4.3	5.8	6.0	7.6	7.5	7.3	6.9	7.3

- How can one obtain an alloy with the desired mix, at the lowest cost. Formulate the problem as a linear programming problem in standard form.
- Propose a way to start the simplex algorithm on this problem.
- Compute the solution with the solver of a spreadsheet program.

4 Collisions

Two geometric objects are defined by the following inequalities:

Object 1

$$\forall i \in [1, 2, \dots, m_1] \quad a_i x + b_i y + c_i z \leq d_i$$

Object 2

$$\forall i \in [m_1 + 1, m_1 + 2, \dots, m_2] \quad a_i x + b_i y + c_i z \leq d_i$$

1. Formulate the linear programming problem allowing one to find out if these two objects are colliding or not.
2. Formulate the dual problem.
3. Here is an example in 2D:

$$\text{Object 1} \begin{cases} x + y \geq 5 \\ -x + 2y \leq 10 \\ 2x - y \leq 4 \end{cases}$$

$$\text{Object 2} \begin{cases} -x + y \geq 6 \\ x + y \leq 10 \\ 2x + y \geq 10 \end{cases}$$

Are these two objects colliding ?